Quantum Field Theory and Representation Theory: A Sketch

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June 17, 2002

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so we have another $S^3 = SU(2)$ internal symmetry to consider.

See [48] for an elaboration of some possible ideas about how this geometry is related to the standard model. There it is argued that the standard model should be defined over a Euclidean signature four dimensional space time since even the simplest free quantum field theory path integral is ill-defined in a Minkowski signature. If one chooses a complex structure at each point in space-time, one picks out a $U(2) \subset SO(4)$ (perhaps better thought of as a $U(2) \subset Spin^c(4)$) and in [48] it is argued that one can consistently think of this as an internal symmetry. Now recall our construction of the spin representation for $Spin(2n)$ as $\Lambda^*(\overline{\mathbf{C}^n})$ applied to a "vacuum" vector. Under $U(2)$, the spin representation has the quantum numbers of a standard model generation of leptons

$\Lambda^*(\overline{\mathbf{C}^2})$	$SU(2) \times U(1)$ Charges	Particles
$\Lambda^0(\overline{\mathbf{C}^2}) = \mathbf{1}$	$(0, 0)$	ν_R
$\Lambda^1(\overline{\mathbf{C}^2}) = \mathbf{C}^2$	$(\frac{1}{2}, -1)$	$\nu_L, \quad e_L$
$\Lambda^2(\overline{\mathbf{C}^2})$	$(0, -2)$	e_R

A generation of quarks has the same transformation properties except that one has to take the "vacuum" vector to transform under the $U(1)$ with charge $4/3$, which is the charge that makes the overall average $U(1)$ charge of a generation of leptons and quarks to be zero.

The above comments are exceedingly speculative and very far from what one needs to construct a consistent theory. They are just meant to indicate how the most basic geometry of spinors and Clifford algebras in low dimensions is rich enough to encompass the standard model and seems to be naturally reflected in the electro-weak symmetry properties of Standard Model particles.

11 On the Current State of Particle Theory

This article has attempted to present some fragmentary ideas relating representation theory and quantum field theory in the hope that they may lead to new ways of thinking about quantum field theory and particle physics and ultimately to progress in going beyond the standard model of particle physics. Some comments about the current state of particle theory and its problems [50, 23] may be in order since these problems are not well known to mathematicians and their severity provides some justification for the highly speculative nature of much of what has been presented here.

For the last eighteen years particle theory has been dominated by a single approach to the unification of the standard model interactions and quantum gravity. This line of thought has hardened into a new orthodoxy that postulates an unknown fundamental supersymmetric theory involving strings and other degrees of freedom with characteristic scale around the Planck length. By some unknown mechanism, the vacuum state of this theory is supposed to be such that low-energy excitations are those of a supersymmetric Grand Unified Theory (GUT) including supergravity. By another unknown mechanism, at even